Applications of sinusoidal phase modulation in temporal optics to highlight some properties of the Fourier transform

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Abstract

Fourier analysis plays a major role in the analysis and understanding of many phenomena in physics and contemporary engineering. However, students, who have often discovered this notion through numerical tools do not necessarily understand all the richness that can be derived from joint analysis in the temporal and spectral domains, particularly in the field of optics. As part of the second year of the master’s degree in Physics Lasers and Materials at the University of Burgundy, we have set up a series of experiments to highlight these concepts and to show, on a non-trivial example of periodic phase modulation, the precautions to be taken in the interpretation of the various experimentally accessible spectra. This hands-on session, made possible through the use of research infrastructure, is also an introduction to the use of standard optical telecommunications equipment.

Keywords: sinusoidal phase modulation, Fourier transform, optical spectrum, signal processing

(Some figures may appear in colour only in the online journal)

1. Introduction

The vast majority of optical instruments such as photodiodes, CCD sensors, or even the eye are sensitive to the intensity of the light wave. Thus, optical telecommunications were initially based on a binary modulation of the intensity of the transmitted wave [1–3]. This made it
possible to neglect to a large extent the factors affecting the phase of the wave, which is much more difficult to control than the intensity profile. However, technological advances in the last two decades have fundamentally challenged this approach by making coherent communications possible, i.e. communications based on the modulation of the optical carrier phase. Then, by supplanting traditional On-Off Keying intensity formats, the advent of coherent communications marked a major turning point in broadband technologies at the dawn of the 21st century [4, 5].

In this contribution, we describe a lab course that aimed to highlight various signatures of the simplest phase modulation possible, i.e. a sinusoidal temporal phase modulation of an initial continuous wave. The temporal waveform of a signal as well as its spectrum (i.e., frequency content) are two fundamental vehicles enabling us to characterize a signal. Following the pioneering work of the French physicist Joseph Fourier [6] who was born in Burgundy 250 years ago, whenever an operation is performed on the waveform of a signal in the time domain (real space), a corresponding modification is applied to the spectrum of the signal in the frequency domain (reciprocal space), and vice versa. Therefore, we want to form an idea of the corollary that the application of a phase modulation in the time domain deeply affects the signal in the frequency domain. We wish to highlight here the possibilities offered by Fourier analysis [7, 8], which plays a major role in countless branches of physics and engineering [9] such as electronics, acoustics, as well as optics [10–13]. While master’s students are used to manipulating this concept in the case of real initial signals (for example, in the case of Fraunhofer diffraction caused by an aperture), we found that they had much more difficulty managing phase-modulated signals. Moreover, throughout this lab session, we wanted to avoid an exclusive numerical approach and we invited our students to manipulate the concept on the state-of-the-art equipment of a genuine research platform where they can benefit from a large set of instruments to directly monitor different consequences of the temporal phase modulation.

The present report is organized as follows. We will first present the laboratory equipment made available to our master’s students as well as the various physical quantities under study, their measurement, and their processing. We then summarize the series of experiments the students had to conduct as well as the key points of each manipulation. Before concluding, we will also discuss how we evaluated the work of our students who followed a master’s degree program in Physics, Lasers, and Materials (French Physics master’s degree taught at the University of Burgundy between 2008 and 2016).

2. Experimental setup and measured quantities

2.1. Research environment

The proposed lab session is based on the PICASSO platform (Innovation and Design Platform for the Analysis and Simulation of Optical Systems, figure 1) at the University of Burgundy. More precisely, this platform, part of the Solitons, Lasers, and Optical Communications team of the CARNOT Interdisciplinary Laboratory of Burgundy, is specialized in high-speed telecommunications and benefits from state-of-the-art light wave characterization equipment. Dedicated to research studies, we planned here for the first time to involve these facilities as part of a pedagogic activity aimed at a master’s level audience (excluding internships in the laboratory). This point was positively noted by our students who highly appreciated using high-end equipment with a higher richness than the devices usually devoted to pedagogy. The interface and the technical documentation of the equipment exclusively in the English language did not seem to discourage the French students who quickly gained a
sufficient level of autonomy for the most basic operations. It should be noted that the technical data sheets summarizing the main characteristics of each device were included in a booklet given to the students well in advance (in this article, we provide the detailed references of all devices so that the reader can easily find the data sheets using any Internet search engine). We also made the students aware of the cost of acquiring such equipment and whether the equipment was mid-range or high-end.

2.2. Equipment made available to the students

2.2.1. Optical source and modulation. The initial optical wave studied in the framework of this lab session consists of a classical optical communication signal. A laser diode (external cavity laser (ECL) technology: diode, model OSICS ECL-1560 manufactured by Yenista) delivers a quasi-monochromatic light beam at a wavelength within the conventional band of optical transmissions. The optical complex field of this initial continuous wave $\psi_c(t)$ can therefore be written as $\psi_c(t) = \psi_0 \exp(i \omega_0 t)$, where $\psi_0$ is the initial amplitude of the wave, and $\omega_0$ is its carrier frequency, which is directly related to its wavelength $\lambda$ by $\lambda = 2\pi/\omega_0$ ($c$ being the velocity of light in vacuum). This signal centered on 1550 nm (i.e. $\omega_0 = 1.216 \times 10^{15} \text{rad} \cdot \text{s}^{-1}$) is then phase-modulated using a lithium niobate (LiNbO$_3$) modulator (Thorlabs, 10 GHz phase modulators, model LN65S-FC). Thanks to the Pockels electro-optic effect, this device converts an incoming electrical modulation into a temporal modulation of the optical phase of the wave [3]. The modulator is characterized by an electrical voltage $V_\pi$, typically 6 V, which corresponds to the external voltage that must be applied to the modulator to induce an optical phase shift of $\pi$. In our setup, the phase modulator is directly powered by an electrical clock delivering a perfectly periodic sinusoidal signal at a frequency of 10 GHz (Anritsu RF/Microwave Signal Generator MG3692B) with an average output power up to 23 dBm. After phase modulation, the optical field becomes $\psi_{m}(t) = \psi_c \exp[i A_m \cos(\omega_m t)]$, where $\omega_m$ is the angular frequency of the sinusoidal modulation, and $A_m$ is the peak deviation of the phase modulation. The temporal phase change $\phi(t)$ induces a change in the instantaneous frequency $\delta \omega(t)$ (relative to the carrier frequency) given by the time derivative of the phase
The use of equipment typical of optical telecommunications makes it easier to handle frequencies in the GHz range, which, as we will see later, is an essential condition for observing an optical spectrum with sufficient accuracy. It should also be noted that all the optical elements used are provided with optical connectors so that no optical alignment is required: the lab session can therefore focus on physical discussions and will not be slowed down by any problems of fine tuning to be carried out. Finally, the use of a light source and modulator with polarization-maintaining elements ensures an increased stability and a very low impact of surrounding conditions.

2.2.2. Temporal analysis. It is rather cumbersome to measure the complete temporal profile of the optical field \( \psi(t) \) directly. Very specific experimental methods such as the frequency resolved optical gating approach \cite{14} exist, but their use remains quite complex and far exceeds the objectives of the course. Visualizing the temporal information contained in the phase requires approaches of an interferometric nature, such as the Michelson or Mach–Zehnder devices. For this lab session, we did not want to involve such devices. Thus, we focused on the detection of the temporal intensity profile alone: \( I(t) \propto |\psi(t)|^2 \). We have a high bandwidth photodiode (>50 GHz, from u2t photonics) connected to an oscilloscope with a suitable electrical analysis bandwidth (33 GHz, Agilent Infinium, DXO-X-93304Q). Since the signal under study is periodic, an optical sampling oscilloscope with a bandwidth of 1 THz (leading to a temporal resolution around 1 ps) is also available in the laboratory (EXFO, PSO-100). The students also had a power meter that allowed them to measure the average power of the light beam. It should be also noted that in order to complement this lab session, essentially dedicated to the spectral analysis of optical signals, the students followed another lab session dedicated specifically to temporal characterization of nano- and picosecond optical pulses. This second set of experiments enabled them to link both domains of analysis, in particular through the issue of bandwidth detection.

2.2.3. Spectral analysis. The main objective of this lab session is to stimulate discussions on the analysis in the reciprocal domain, i.e. the use of the Fourier transform. The expression of the field \( \hat{\psi}(\omega) \) in the spectral domain is thus given by the following Fourier transform (with \( F \) denoting the Fourier transform):

\[
\hat{\psi}(\omega) = F(\psi(t)) = \int_{-\infty}^{\infty} \psi(t) \, e^{i \omega t} \, dt.
\]

Similarly to the time domain, it is experimentally difficult to access the amplitude and phase of \( \hat{\psi}(\omega) \). Thus, with conventional devices, only the spectral intensity profile \( S(\omega) \propto |\hat{\psi}(\omega)|^2 \) is easily recordable. This parameter represents the optical spectrum, which is a measure commonly used in the field of spectroscopy. The students had to test two different optical spectrum analyzers (OSAs) and compare the results obtained. The resolution of the first OSA was 0.07 nm (mid-range equipment, Anritsu MS9710B; denoted here as OSA1) and that of the second one was <0.1 pm (high-end equipment, APEX AP2441B; denoted here as OSA2).

As far as frequency analysis is concerned, we did not want to limit ourselves to the optical spectrum. Thus, we also studied the electrical spectrum (also known as the radio frequency spectrum), which corresponds to the Fourier transform of the intensity temporal profile:

\[
R(\omega) \propto F(I(t)) \propto \int_{-\infty}^{\infty} |\psi(t)|^2 \, e^{i \omega t} \, dt.
\]

We provided students with an electrical spectrum analyzer (ESA) with a 26 GHz bandwidth (Agilent EXA Signal Analyzer, N9010A). To benefit from the highest measurement...
dynamics, we chose a dedicated device but the students also discovered the mathematical functionalities now routinely implemented in the latest generations of real-time oscilloscopes.

2.2.4. Optical linear spectral shaping of light. The last step of the session involves testing the impact of a change in the phase or amplitude properties of the optical spectrum. We compared two optical elements. The first one was a simple single-mode optical fiber. In this case, the dispersion of the fiber imprints a spectral quadratic phase that is easily modeled \[ \psi_{\text{out}}(\omega) = \psi_{\text{in}}(\omega) \exp \left( i \frac{\beta_2}{2} L (\omega - \omega_0)^2 \right) , \] (3) with $\beta_2$ being the dispersion coefficient of the group velocities, and $L$ being the length of the fiber. We used a spool of the most standard fiber in the telecom industry, the SMF-28 fiber. Its properties are normalized according to recommendation G.652 of the International Telecommunication Union [16]. The dispersion coefficient of this fiber is $\beta_2 = -20 \times 10^{-3}$ ps$^2$ m$^{-1}$ and its length is 2 km.

To go beyond a simple parabolic change of the spectral phase, we also provided students with a programmable optical shaping device based on a method similar to the 4-f method [17]. Previously restricted to only a few laboratories and femtosecond applications [18], linear optical shaping has now emerged in the wavelength domain of optical telecommunications with easy-to-use commercial devices [19] (we use a Finisar Waveshaper 1000S device). The principle of this spectral processing is illustrated in figure 2. The shaper is able to impose a spectral phase and amplitude transfer function $T(\omega)$ such that $\psi_{\text{out}}(\omega) = T(\omega) \psi_{\text{in}}(\omega)$. In this lab session, we limited ourselves to the use of simple transfer functions directly embedded in the control software, such as the addition of a quadratic phase $T(\omega) = \exp \left( i D (\omega - \omega_0)^2 \right)$ (with $D$ being a positive or negative coefficient) or amplitude filtering by a Gaussian shape whose central frequency $\omega_1$ and bandwidth $\omega_2$ can be varied: $T(\omega) = \exp \left( - (\omega - \omega_1)^2 / \omega_2^2 \right)$.

The consequences of this signal processing can be efficiently predicted using numerical simulations. Since the carrier frequency $\omega_0$ is several orders of magnitude higher than the modulation frequency $\omega_{\text{in}}$, it is relevant to neglect the rapid oscillations of the optical carrier and to get interested in the evolution of the slowly varying envelope exclusively. Therefore, the first step is to remove the carrier by considering $\psi_{\text{in}}(t)$ multiplied by $\exp(-i \omega_0 t)$. The

![Figure 2: Principle of linear spectral phase or amplitude shaping.](image-url)
resulting envelope \( \psi'_{\text{in}}(t) \) is then sampled using \( 2^n \) samples (\( n \) being an integer), and \( \widetilde{\psi}_{\text{out}}(\omega) \) is obtained thanks to a fast-Fourier transform (FFT) algorithm that is implemented in all major scientific packages. The transfer function that should be applied is \( T'(\omega) \) (which is simply \( T(\omega) \) frequency offset by \( \omega_0 \) : \( T'(\omega) = T(\omega - \omega_0) \)) so that the spectral content \( \widetilde{\psi}_{\text{out}}(\omega) \) after processing becomes \( \psi'_{\text{out}}(\omega) = T'(\omega) \psi'_{\text{in}}(\omega) \). The temporal envelope \( \psi'_{\text{out}}(t) \) is then retrieved thanks to an inverse FFT.

Note that in order to compensate for the optical losses induced by the spectral shaper, we can take advantage of the optical gain provided by an erbium-doped fiber amplifier, which has now become a standard and essential device in the telecommunications industry, used as an optical repeater in long-haul transmissions [1, 20, 21]. Indeed, when pumped with a laser at a wavelength of 980 nm or 1.48 \( \mu \text{m} \), a silica fiber doped with erbium ions provides gain in the 1.55 \( \mu \text{m} \) region, which corresponds to the spectral window where fiber losses are minimal. This amplification is achieved by stimulated emission of photons from dopant ions. However, spontaneous emission may also exist and decrease the quality of the amplified signal.

**2.3. Complete laboratory setup and alternative solution**

The complete setup is presented in figure 3(a), where we have distinguished using different colors the parts dealing with an electrical signal from those dealing with an optical signal. The
practical implementation does not present any particular difficulty, the whole setup being fibered [6]. Thus, the students themselves handled and connected the various devices as the lab session went on. This lab session was an opportunity to make our students aware of the precautions to be taken when handling optical connectors (prior cleaning, laser risks) or RF electrostatic-sensitive devices. In particular, we stressed the importance of impedance matching conditions and electrostatic shocks. In total, the equipment used in this experiment represents an acquisition budget of approximately 500 k€ (mainly for detection equipment, the optical sampling oscilloscope, the real-time digital oscilloscope, and the high-resolution optical spectrum analyzer, costing more than 100 k€ each). Such a cost made it crucial that we cooperate with a platform dedicated to research. Such a collaboration contributes to the attractiveness of the academic course in experimental sciences.

For master’s students who cannot have access to such research facilities, there are alternatives using much less expensive equipment, as illustrated in figure 3(b), i.e. using devices routinely available in optics or electronics labs. The 10 GHz clock can be replaced by inexpensive amplified radio frequency oscillators operating at a higher frequency (around 22.24 GHz in the range typical of the K band of the microwave communications). At this modulation frequency, spectral measurements become relevant on the widely available OSA1: the spacing between the spectral discrete components that, as we will see, emerge in the spectrum will not be too heavily impaired by the limited spectral resolution of this OSA. Furthermore, in order to lower the requirements of the bandwidth of the temporal detection, it is possible to operate the spectral phase shaping at a lower frequency (around 5 GHz) by involving a longer piece of optical fiber or a fiber with a higher dispersion. In this context, widely available research oscilloscopes and RF spectrum analyzers (that can also be purchased on the second-hand market) will be sufficient to qualitatively observe the temporal changes experienced by the intensity profile. Therefore, the approach we here describe using state-of-the-art components can accommodate a much more affordable setup and can be implemented in many academic institutions with master’s programs in the field of optics.

We have summarized in figure 4 the various quantities that the students had to experimentally deal with during this lab session. Since the coherent signal under study is
periodically modulated with a period \( T = \frac{2\pi}{\omega_m} \), the optical spectrum exhibits a comb structure with spectral lines equally spaced by \( \omega_m \): it is made of discrete components at frequencies \( \omega_n = \omega_0 + n \omega_m \) (\( n \) being an integer) with a level \( S_n = S(\omega_n) \). Note that the periodicity of the temporal modulation only presents visible signatures on the RF spectrum when the temporal intensity profile is modulated.

3. Description of the experimental work

The practical session took place in three subsequent stages, with the complexity of the experiments gradually increasing. The students had to first think over the spectral analysis of a continuous signal before studying the various consequences of the temporal phase modulation. The last step was to manipulate the basic spectral properties of the resulting signal.

3.1. Analysis of a continuous optical wave

To begin this experimental work and get to grips with the different diagnostic elements, we first studied the spectral properties of the optical signal emitted by the ECL diode. The students had to determine the spectral linewidth at half maximum of the diode and the noise level of the component. These \textit{a priori} simple characteristics nevertheless require a good understanding of the settings of the optical spectrum analyzer. It is therefore important to choose the finest resolution. For OSA1, the resolution varies between 0.07 and 1 nm. Comparison of the measurements shows that the resolution is crucial for the shape of the recorded signal (figure 5(a)). None of the proposed resolutions (0.07 nm, 0.2 nm, and 0.5 nm) allow us to conclude the spectral width of the source; the technical documentation of the laser indicates a value of 150 kHz (i.e. \( 1.2 \times 10^{-6} \) nm). With the higher-end model (resolution of only 5 MHz, figure 5(b)), students can infer that the source has a spectral width of less than 5 MHz, which is in accordance with the technical documentation. Therefore, for the highly coherent source we use, and given the resolution of the current OSA, it is not possible to provide a value of the spectral linewidth by a direct measurement in the optical domain.

Regarding the noise level measurement, a good understanding of the technical data is once again necessary. Indeed, depending on the settings used for the instrument and its acquisition mode, the measurements can be limited by the electronic acquisition noise. The students therefore had to think over the nature of the noise floor recorded on the OSA, i.e. the limit of the optoelectronic detection or noise that can be attributed to the tested component.
From the measurements made with OSA2, some spurious sidebands symmetrically located with respect to $\omega_0$ are visible and are typical of residual longitudinal modes of the cavity. Their level below $-50$ dB is consistent with the >45 dB side mode suppression ratio provided in the component data sheet.

To complete this first step of the experiment, we studied the signal recorded by the RF analyzer. The students had to check that in the basic configuration the RF spectrum did not provide any relevant measurements of the spectral width of the diode. Unlike the optical analyzer, which gave important information about wavelength (i.e. the carrier frequency $\omega_0$), the RF spectrum is only sensitive to the intensity profile of the envelope of the wave. However, let us mention that more advanced heterodyne architectures can provide very accurate access to the diode linewidth.

3.2. Spectral analysis of the phase-modulated light

The core of this lab session lies in the spectral analysis of the phase-modulated signal. The students now operate the phase modulator by supplying it with a varying sinusoidal voltage at a frequency of 10 GHz and with an average power of 20 dBm. Contrary to in optical intensity modulators, no bias voltage is required here.

3.2.1. Analysis in the RF spectral domain and in the temporal domain. Students started by observing the RF spectrum of the phase-modulated signal. Searching for a signature of the modulation, they increased the sensitivity to the maximum of the ESA and succeeded in detecting an extremely weak line emerging slightly from the noise at the frequency $\omega_m$. Nevertheless, they were surprised to find that when the ECL diode is switched off, this low peak remains: it cannot therefore be characteristic of the optical wave. It is in fact due to the electromagnetic radiation from the generator and the unshielded cables and connectors that are involved in the setup. Optical phase modulation therefore does not impact the RF spectrum, unlike phase modulation as traditionally used in the field of frequency modulation-based radio or microwave communications [22, 23]. In the latter case, the RF spectrum analyzer allows us to see the impact of modulation on the carrier frequency, and is not restricted, as in the case of optics, to information on the envelope. To confirm that there was no impact on the envelope intensity profile, the students were able to visualize the signal on the fast oscilloscopes at their disposal. They also checked that the presence of the phase modulation was not involving additional power losses on the optical signal (except for the insertion losses of the component).

3.2.2. Analysis in the optical spectral domain. The students then observed the optical spectrum of the signal. They first used OSA1. In contrast to the RF analysis previously carried out, a very significant change in the optical spectrum is observed in the presence of modulation (figure 6). Even if the central wavelength does not change ($\omega_0$ is not affected), the resulting spectrum is now very different from an isolated (even widened) peak characteristic of a continuous signal. The spectrum is symmetrical and has a significant width and a complex structure with regularly spaced bumps. By changing the amplitude of the electrical (and therefore optical) modulation, the students verified that the width and shape of the spectrum varies according to $A_m$ while the intensity profile $I(t)$ remains unchanged. This point is an apparent violation of a simplified relationship on which many students base their qualitative thinking: for a Fourier transform-limited waveform, the wider a spectrum, the shorter the time structure. As we can see with this simple example, for the complex light field, this relationship must be handled with care.
Since the spectral resolution of OSA1 (0.07 nm or 8.7 GHz) is very close to $\omega_m$, the use of an OSA with a better resolution is essential to make unambiguous observations, in particular to reveal the fine details of this structure and to make a quantitative link with analytical theory. Examples of the results obtained with a high-end optical device (OSA2) are shown in figure 7. First of all, these recordings allow us to clearly see a spectrum of lines that was partially obscured by the limited resolution of OSA1. This comb nature is in perfect agreement with the periodic nature of the signal under test. Consequently, the spectral analysis of the signal becomes limited to the analysis of a set of discrete lines. By setting the
OSA display in frequency units rather than in wavelength units, the students were able to check that the frequency spacing between two lines corresponds exactly to the modulation frequency $\omega_m$ of the signal: by modifying $\omega_m$ on the electrical generator, they can straightforwardly reduce or increase the spectral spacing without changing the overall shape of the spectrum. We also note the symmetrical nature of the experimental spectrum, the decreasing amplitude of the components (for the range of $A_m$ under study and excluding the central component), as well as the excellent signal-to-noise ratio of the optical signal. The students greatly appreciated the great measurement dynamics offered by the OSA devices, which are capable of accurate measurements over more than six orders of magnitude.

By changing the power of the electrical signal, i.e. $A_m$, we observe an increase in the extent of optical spectrum. Thus, the value of the measured spectral components varies. The link between these experimental recordings and the theory is quite simple and can benefit from a Jacobi–Anger expansion [24, 25] of the modulated field:

$$\psi_m(t) = e^{i\omega_m t} \sum_{n=-\infty}^{\infty} i^n J_n(A_m)e^{i\omega_n t}.$$  

(4)

The amplitude $\tilde{\psi}_m$ of the spectral component at $\omega_m = \omega_0 + n\omega_m$ is therefore analytically predicted from Bessel functions $J_n$ of the first kind and is of (integer) order $n$:

$$\tilde{\psi}_m \propto i^n J_n(A_m).$$  

(5)

Bessel functions, which are special mathematical functions [26, 27], are nowadays implemented in most scientific computation and programming software. We recently published an article interpreting this result qualitatively as a consequence of a two-wave interference process [28]. Bessel functions appear in many different problems of wave propagation and static potentials, ranging from the study of the far-field diffraction through a circular aperture [29] to the vibration of a cantilever beam or a circular membrane [30]. One of our students commented that the spectral pattern reminded him of the diffraction he studied during an internship dealing with diffraction of sinusoidal phase gratings. This is no coincidence since diffraction can be modeled using similar mathematical treatments and is consequently equivalent, leading to an exciting space–time duality [31].

The amplitudes of the Bessel functions are plotted in figure 8. Formula (5) highlights several properties illustrated in figure 6 that can be confirmed experimentally. First of all, the amplitude of the lines does not depend on the modulation frequency $\omega_m$ but only on the modulation amplitude $A_m$. It is possible to have, for a certain modulation amplitude, $J_0(A_A) = J_1(A_A)$ (point A, obtained for $A_A = 1.435$ rad) or $J_2(A_B) = J_0(A_B)$ (point B, $A_B = 1.841$ rad) or even $J_0(A_C) = 0$ (point C, $A_C = 2.405$ rad). Then, an important point to notice in equation (5) is the factor $i^n$ that leads to a phase offset of $\pi/2$ between two successive spectral components; the spectral components are not in phase and thus the spectrum is not Fourier transform-limited [32].

However, the optical spectrum made of line $S_m$ is the square modulus of the Fourier transform of the field:

$$S_m \propto |J_n(A_m)|^2.$$  

(6)

Therefore, it is not possible to visualize the relative sign change between the different components. By performing a series of measurements as a function of the electrical power, the students could plot the evolution of $S_m$ according to $V_m$ (figure 9(a)). It should be noted that the students had to convert the average modulation powers into voltage by involving, in particular, the characteristic impedances of 50 $\Omega$ of the RF link. To compare the predictions of equation (6) and the experimental measurements, we also must convert the value of the
electrical modulation $V_m$ into optical modulation $A_m$. It is then interesting to look at the properties of point A, whose characteristic voltage can be measured with good accuracy. Indeed, for 2.51 V, we obtain a modulation amplitude such that $J_0 = J_1$, i.e. $A_A = 1.435$ rad. This therefore imposes the proportionality coefficient between the two quantities, which is 0.57 rad/V. We can deduce an experimental value of the coefficient $V_\pi$ characteristic of the modulator $V_\pi = 5.5$ V, which is in agreement with the manufacturer’s technical documentation. Point B or C could also be used, and lead to very similar values for $V_\pi$. Figure 9 shows the superposition of the experimental measurements and those expected from equation (6). The agreement obtained for the central spectral lines and the five lateral sidebands is

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**Figure 8.** Behavior of the Bessel functions of the first kind with order from 0 to 3.

**Figure 9.** Evolution of the sidebands $S_n$ according to the maximum voltage $V_m$ applied to the phase modulator. (a) Results plotted on a linear scale for the central sideband and the first three sidebands. (b) Results plotted on a logarithmic scale for the central and the first five lateral sidebands. The various solid and dashed lines represent the analytical predictions based on Bessel functions of the first kind (equation (6)).
absolutely remarkable, even when displayed on a logarithmic scale. This proves the quality of the temporal phase modulation scheme as well as the quality of the spectral detection. From the experimental measurements, it can be observed that for an electrical power of 22.7 dBm, the central component is nearly completely vanished, with an extinction ratio close to 40 dB (see also the spectral record at this power on figure 7). This set of measurements also confirms that the central components and the first lateral sideband do not monotonously behave with the amplitude modulation: after point C, the central component increases and we can note the maximum level reached by the first pair of sidebands when a voltage of 3.3 V is applied to the phase modulator.

3.3. Modification of the spectral properties.

In the final part of this lab session, our objective was to modify the spectral properties of the phase-modulated signal. This constitutes an introductory approach to optical linear spectral processing.

3.3.1. Use of a single-mode fiber spool. The first method we exploit is extremely basic. We implement a standard optical fiber spool placed just after the modulator. Thanks to the power meter, students can first check that the attenuation in 2 km of fiber is very moderate, below 1 dB, including fiber connectors. The optical spectrum shows absolutely no change in the shape of the spectrum, keeping its frequency comb structure unaltered. Students therefore do not expect to observe any other changes, either on the ESA or on the oscilloscope. Indeed, for many of them, in the usual regime of linear propagation, the fiber stands as a communication channel that does not affect the intensity of a continuous signal. For the students, a phase change should not affect the measured temporal aspect. Nevertheless, their observations of the RF spectrum invalidate their first intuition. Indeed, as shown in figure 10(a), they detect a peak at the modulation frequency $\omega_m$. This very narrow peak is now perfectly discernible from the noise and is not due to electromagnetic compatibility concerns. The students are also surprised to see a second peak at the frequency $2\omega_m$ (figure 10(b)). Since the bandwidth of our ESA is limited to 26 GHz, the detection of additional harmonic peaks cannot be performed.

If the RF spectrum is modified, it is implied that the temporal profile of the wave intensity is affected; one cannot go without the other. The presence of at least two frequencies indicates that the corresponding profile is more complex than a simple sinusoidal modulation of the...
temporal intensity profile. To visualize this change, we used fast oscilloscopes. Most students were then surprised to find that the initially continuous intensity profile changed into a pulse profile lying over a continuous background (figure 11(a)). The digital oscilloscope provides a full width at half maximum duration around 16 ps. Due to the optoelectronics bandwidth limitation, this value is slightly higher than the duration of 15 ps provided by the optical sampling oscilloscope. The repetition rate is dictated by the modulation frequency so that the frequency of the optical train is directly adjustable by the electrical signal generator. This reshaping of the temporal intensity highlights the different but symmetrical roles played by the temporal and spectral phases. If modifying the temporal phase impacts the spectral intensity profile without affecting the time intensity profile, the opposite happens when the spectral phase is modified.

The experimental observation can be quantitatively reproduced (figure 11(b1)) using the approach detailed in section 2.2.4 and summarized in figure 2. The agreement is excellent in terms of waveform and temporal duration as well as regarding the ratio between the peak power of the periodic structure and the continuous background. For a qualitative understanding of the process stimulating the emergence of those temporal structures, one

Figure 11. Temporal intensity profile after propagation in a 2 km long optical fiber. (a) Experimental results. (b1) Results from numerical simulations. The dashed horizontal line represents the intensity profile monitored directly after temporal phase modulation. Temporal intensity profiles are normalized with respect to the background level. (b2) Instantaneous frequency obtained after phase modulation. The various gray arrows indicate the impact of the fiber dispersion. The amplitude of the initial phase modulation \( A_{\text{m}} \) is around 2 rad, and \( \omega_{\text{m}} = 10 \text{ GHz} \).
3.3.2. Use of a spectral phase shaper. We then led the students to use a spectral shaper. At first, we only used phase shaping features. In particular, the shaper allows the inclusion of an additional spectral parabolic phase [40]. By playing on the parabolic phase level and using $D = \beta_2 L/2$, the students were able to reproduce the results previously obtained with a fiber. The students were also able to see that the sign of dispersion for this application does not matter. However, a limit appears for strong phases for which attenuation of certain spectral components may be visible.

The use of a spectral shaper as dispersion emulator [40] confirms that a fiber is nothing but a quadratic phase filter [15]: The fiber retains some advantages such as its low cost and its very low level of attenuation, which is much lower than that introduced by linear shaping (4 dB of loss related to the component). However, the shaper has the advantage of flexibility. It allows the value of the dispersion to be precisely adjusted to obtain the shortest pulse train. In a more advanced architecture, fine tuning of the spectral phase of a limited set of spectral lines can also enable generation of parabolic, triangular, or rectangular intensity profiles [41]. This has stimulated strong practical interest for applications in the field of microwave photonics where manipulating light can help generate electrical waveforms that would be not possible with the current bandwidth limitations of electrical systems [42].

3.3.3. Use of a spectral intensity shaper. Finally, the students tested a spectral intensity shaping, i.e. applying a bandpass optical filter to the signal by means of the previous waveshaper. We asked them to compute a Gaussian bandpass filter with a full width at half maximum of 25 GHz, offset by 25 GHz from the carrier frequency. The programming is very simple and the students were able to visualize directly on the OSA the result of this filtering, which eliminates a number of components and reduces the amplitude of other components. We asked the students to experimentally check that the spectral filter corresponded to our expectations. This question was not so trivial for them. Indeed, the simplest approach is to
basically compute the ratio between the output signal and the input signal. But the comb nature of the spectrum slightly complicates the analysis, particularly because of the detection noise between the spectral lines, which disturbs the ratio. As shown in figure 12, students must only take into account the level of the narrow spectral lines and then fit their data by a Gaussian shape to verify that the experimental points are in agreement with the target.

The students also noted that this adjustable filter did not allow the precise generation of filtering profiles with a width below 10 GHz without adding a noticeable level of losses. Regarding the temporal intensity profile, we asked our students what would be observed if our optical bandpass filter were thin enough to isolate a single spectral line shifted by $\omega_m$, $2\omega_m$, or $3\omega_m$. Many students spontaneously answered a sinusoidal evolution of the intensity profile at the angular frequency $\omega_m$, $2\omega_m$, or $3\omega_m$. The correct answer is rather different, and the students once again confused the frequencies affecting the carrier and the envelope. Indeed, by isolating a single spectral line, we get a continuous wave, whose carrier frequency has been frequency offset. This opens a very exciting perspective for wavelength multiplexed communications: for example, isolating the spectral lines of a supercontinuum source has enabled the generation of more than 1000 independent transmission channels [43]. Similar motivation currently drives the field of optical microresonators [44]. In both cases, the initial comb is not generated thanks to a phase modulator, but results from the self-phase modulation that affects any high-power pulse propagation in a nonlinear optical waveguide [45].

When monitoring the average power on the power meter, a number of students were surprised by the rather high level of losses observed following the intensity filtering: more than 10 dB had simply disappeared. After reflection on their part, they made the link to the suppression of spectral components, which is accompanied by a loss of energy. Unlike advanced nonlinear shaping techniques [46], linear intensity shaping is an intrinsically dissipative operation that does not allow any redistribution of energy between the different components. This significant loss of energy makes the detection on the oscilloscope quite noisy. We then include in the assembly an erbium-doped fiber amplifier that allows us to regain the lost energy. Nevertheless, students notice an increased in the noise level on the

Figure 12. (a) Optical spectrum (a1) before and (a2) after the optical bandpass filter. (b) Ratio between the input and output levels of the various spectral lines (diamonds) and the corresponding fit by a Gaussian waveform. Results are plotted on a logarithmic scale and the optical frequencies are provided with respect to the carrier frequency $\omega_0$. Results are obtained for a power of 21 dBm leading to $A_m = 2$ rad. Optical frequencies are relative to the optical carrier $\omega_0$. 
spectrum, with the stimulated amplification accompanied by the spontaneous noise emission. A discussion then allows them to understand that it is preferable to place the amplifier before the spectral shaper because the latter will help eliminate the noise brought in.

The last part of this work deals with the experimental observation of the resulting temporal signal on the oscilloscope. The students then observe a pulse train at a frequency corresponding to the initial phase modulation (figure 13(a)). By tuning the wavelength offset of the filter, they can see that the farther away the filter is from the central line, the more the generated pulses are isolated and short. Compared to the phase spectral shaping previously discussed, the duration of the resulting pulses is longer, but the resulting signal has the advantage of not presenting any background. Students find strong marks of this periodicity on the RF spectrum with significant lines at 10 GHz and 20 GHz, confirming that the signal is not a pure sinusoid. A frequency offset optical bandpass filtering is therefore a second convenient solution to convert the phase modulation into a periodic pulse train. This solution is also used in [47] as well as in telecommunications [48–50]. Again, it should be noted that such a behavior does not appear natural for students that are used to the fact that when the spectral width is reduced the duration should increase. Here, it is the opposite, and the limitation of the spectral range has made it possible to create shorter structures.

Once again, and as shown in figure 13(b2), the students were able to compare their experimental observation with the numerical simulations based on the approach described in section 2.2.4. It is the notion of instantaneous frequency $\delta \omega$ that will allow us to understand this phenomenon qualitatively. It should be recalled that in the previous method, the parabolic spectral phase made it possible to converge energy at specific times without loss of energy. With wavelength-shifted filtering, the approach is different and inherently dissipative. As illustrated in figure 13(b1), the bandpass filter only keeps the frequencies corresponding to one extrema and strongly attenuates the other components. The energy localized in time slots for which the instantaneous frequency is different from the central frequency of the filter vanishes. In contrast, for time slots where the instantaneous frequency is close to the central frequency of the filter, the energy is conserved, leading to the emergence of light pulses.

Figure 13. (a) Experimental recording of the pulse train obtained after frequency offset optical bandpass filtering. (b) Numerical simulations of (b1) the instantaneous frequency and (b2) the temporal intensity profile. The shaded area represents the instantaneous frequency that is suppressed by the optical bandpass filter. Results are obtained for an RF power of 21 dBm leading to $A_m = 2$ rad.
4. Assessment of students’ results

This laboratory session follows an eight-hour lecture detailing the various parameters characterizing the temporal, spectral, and spatial properties of light and introducing all the theoretical concepts required for a good understanding of the laboratory session, which lasted half a day (i.e. between 3 and 4 h). A detailed description of the work expected during this lab session and all the technical information of the experimental devices was made available well in advance to the students. Therefore, the students were asked to prepare for the laboratory session by carefully reading its detailed description and trying to answer several preliminary questions that were directly linked to the lecture. The experiments took place in the presence of a teacher or researcher. This supervision is justified by the high cost and specificity of the equipment made available to the students and by our will to establish a live interaction throughout the lab session regarding the various experimental observations. We noted the need for numerous reminders to correct erroneous pictures that students could have in mind, for example, about the optical phase or, more generally, about the spectra they recorded. The students worked in pairs or trios. During this session, they were evaluated on how seriously they worked as well as on their level of personal involvement (15% of the final mark), our goal being to avoid any passivity. Since we wanted to stimulate exchanges, we did not take into account the errors made or the questions asked.

Following the lab session, the students had to write a detailed report of their work (one report per pair or trio). They had several weeks to complete this report and we encouraged them to exchange information between different groups as needed. The quantitative analysis of the experimental data they accumulated during the session was very time-consuming for some groups. This highly demanding task may have required up to 8 h of personal work to get a full and crystal-clear picture of the proposed experiments. Students were also strongly encouraged to use software such as Matlab® or its clones to numerically reproduce and quantitatively validate their experimental observations. No ready-to-use code was provided to them since their experience in other teaching modules allowed them to be autonomous in this task.

The next step was to simulate for our master’s students the review process for peer-reviewed journals. Thus, the various reports were exchanged between the teams and everyone had to assess the strengths and weaknesses of their classmates. The teams then had the opportunity to respond to these comments and modify their work accordingly. The reports were evaluated by the teaching team both before and after the reviewing process (30% and 15% of the final mark, respectively). The relevancy of the review of the other team was also evaluated (15% of the final mark). Regarding the assessment of the final report, the evaluation focused on how accurately the different comments raised by the review were answered and how it translated into improvements of the manuscript.

The final reports corrected by the teachers were given back to the students before the last stage of the evaluation process. This consisted of a short oral interview (about ten minutes long) to determine the overall understanding of the main theoretical and experimental concepts discussed in the framework of this lab session. This last step was carried out individually and was worth 25% of the mark.

5. Conclusions

This lab session was carried out over four years, from 2012 to 2016, with a total of about 40 students participating. The feedback was very positive, both from the master’s students, who were delighted to handle state-of-the-art equipment on a concrete problem, and from the
supervisors, who strongly appreciated the personal investment shown by a few teams. The sessions provided a real dialogue around the importance of the optical phase, telecommunications, and the concept of optical and electrical spectra, thus correcting a number of misconceptions that students had. This practical work clearly helped the students better understand the usefulness and complementarity of the different analyses that can be carried out on a signal and the nuances between electrical and optical spectra. In contrast to the general way of thinking of our students, that the spectral characterization in the electrical and optical domains are basically similar, here the present lab session enables them to highlight the advantages and drawbacks of each domain, in particular the greater precision of the RF spectrum obtained in the lowest frequencies. We showed the students that even a modulation of phase as simple as a sinusoidal waveform can be relevant for many modern applications of fiber optics [31, 33–35, 39, 47, 51, 52]. The students also found the discussion helpful for understanding nonlinear effects, and more specifically why the self-phase modulation induced by a high-power pulse leads to a spectral broadening [51, 53, 54]. We found that the practical examples displayed directly on the measuring devices can be a very valuable complement to the numerical simulation tool by reducing the potential passivity of some students. Finally, these lab sessions also enabled us to discuss academic research currently under investigation by different groups. This shows that the analysis proposed by Joseph Fourier [55, 56] (of whom 2018 marks the 250th anniversary of his birth in our region) remains of the most crucial importance in modern applications such as high-speed transmissions.

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