We report numerical and experimental studies of multiple four-wave mixing processes emerging from dual-frequency pumping of a passive nonlinear fiber ring cavity. We observe the formation of a periodic train of nearly background-free soliton pulses associated with Kerr frequency combs. The generation of resonant dispersive waves is also reported.

Optical frequency combs in synchronously pumped passive Kerr resonators have attracted considerable interest during the last decade as a complementary approach to comb formation based on mode-locked lasers. Microresonator frequency combs, particularly based on CMOS-compatible platforms, may provide compact and energy-efficient coherent multi-wavelength sources for optical communication and spectroscopy applications. The formation of Kerr combs usually arises through modulation instability (MI) and cascaded four-wave mixing (FWM) processes [1,2]. Such nonlinear dynamics has been extensively studied in optical fibers to generate high-repetition-rate pulse trains for almost 30 years. Several different pumping configurations can be investigated [3,4], for example, single continuous wave (CW) pumping with or without a weak seed (seeded MI), and bichromatic pumping implemented either by using two separate CW lasers of equal power or by modulating a single CW laser. The latter approach directly involves multiple four-wave mixing processes, and the pulse repetition rate can be simply determined by the frequency of the initial beating. The nonlinear compression associated with multiple FWM processes taking place within an anomalous dispersive optical fiber has proven to be an attractive and efficient method to generate very high-repetition-rate pulse trains and broadband frequency combs by combining stability with the simplicity of the experimental setup [5,6]. In the case of resonant cavities, most previous works have focused on monochromatic CW pumping, either with or without parametric seeding. However, several theoretical studies have demonstrated the benefits of the dual pumping configuration [7–9], for example, for the generation of frequency combs without a pump intensity threshold and the possible emission of a periodic train of background-free soliton pulses. Recent experimental works in microresonators have confirmed the thresholdless comb generation process and its potential for frequency-degenerate parametric oscillation via dual pumping [10,11]. More recently, a dual-pump approach has proven successful for generating robust frequency combs with a varying free spectral range spacing (FSR) in a microresonator [12]. However, in that case, the dual pump self-oscillates in the laser cavity loop thanks to a fiber-microresonator dual-cavity architecture, rather than by coupling external CW pumps. This mode-locking method is known as filter-driven or dissipative FWM [13,14].

We report here on an extensive numerical and experimental study of the dual-pump configuration in a passive nonlinear fiber ring cavity, which confirms the presence of a rich variety of different comb states as the pump intensity and cavity detuning are changed. Complementary spectral and temporal characterizations are performed to reveal the corresponding pulse patterns associated with dissipative FWM-induced Kerr combs.

First, we provide an overview of the predicted comb states from numerical simulations, where the nonlinear fiber ring cavity with anomalous dispersion is bichromatically pumped. Our resonant ring cavity consists of a 26.5 m long segment of highly nonlinear optical fiber (HNLF) [15]. The fiber combines a low and flattened group velocity dispersion with a high nonlinearity ($\beta_2 = -0.89 \text{ ps}^2 \text{ km}^{-1}$, $\gamma = 10 \text{ W}^{-1} \text{ km}^{-1}$) at 1552.4 nm. A 90/10 input coupler is used to close the fiber loop cavity, whereas a 99/1 output coupler permits us to extract and characterize the intracavity field. The two couplers are made of SMF28 fiber with a total length of 1.5 m that belongs to the cavity (i.e., the total cavity length is equal to 28 m).

Our modeling is based on solving the following Ikeda map [16]:
\[
\frac{\partial A^m(z, t)}{\partial z} = -i\beta_2 \frac{\partial^2 A^m(z, t)}{\partial t^2} + i\gamma |A^m(z, t)|^2 A^m(z, t),
\]

(1)

\[
A^{m+1}(0, t) = \sqrt{1 - \alpha A^m(L, t)} \exp(i\phi_0) + \sqrt{R}A_{in}(t),
\]

(2)

where \(A_m\) represents the envelope of the intracavity optical field at the \(m\)th round-trip in the cavity, and \(A_{in} = (2P_0/\alpha)^{1/2} \cos(\Omega t)\) is the external bichromatic pump field (\(\Omega\) being the frequency spacing between the two pumps, and \(P_0\) being the total average power). Both \(A_m\) and \(A_{in}\) are expressed in a reference frame moving with the group velocity at the carrier frequency \(\omega_0\). \(z\) and \(t\) represent the propagation distance inside the cavity and the time in this reference frame, respectively.

\(L\) is the cavity length and \(R\) represents the reflection coefficient of the coupler closing the cavity. In the following, we will refer to the normalized detuning of the cavity, defined as \(\Delta = 2(2\pi k - \phi_0)/\alpha\), where \(k\) designates the nearest resonance, \(\phi_0\) designates the associated phase detuning of the dual pump (i.e., a symmetrical detuning of the two pumps is considered), and \(\alpha\) represents the total power lost per round-trip.

With a proper choice of pump power and cavity detuning, we may expect to be able to generate a periodic train of nearly background-free soliton pulses associated with Kerr frequency combs [7]. Figure 1 reports examples of the distinct simulated regimes in the spectral domain as a function of detuning (\(\Delta\)) and pump power (\(P_0\)). These simulations also indicate the temporal profile of the generated pulse trains and its corresponding autocorrelation trace for easy comparison with experiments. Here, the frequency spacing between the two pumps was fixed to 80 GHz. The first two cases [Figs. 1(a) and 1(b)] and Figs. 1(c) and 1(d)] show the formation of a train of stable multi-pulse structures inside the cavity (two and four pulses, respectively). Spectrally, these structures correspond to Kerr frequency combs formed by multiple FWM, with an amplitude modulation due to MI gain bands. Cavity detuning has a key role in phase-matching the MI [16]. In general, the higher the value of detuning is, the higher must be the value of the input power in order to maintain the MI. By increasing the cavity detuning, another stationary state of regular pulse train is achieved, which is associated to a triangular spectrum [Figs. 1(e) and 1(f)]. The pulse duration is of the order of 850 fs. For a detuning corresponding to the bistable regime of the cavity [Figs. 1(g) and 1(h)], we observe a train of cavity soliton-like pulses (whose spectrum is well fitted by a 415 fs sech pulse) driven by the modulated temporal field (instead of the CW field) [17]. Note that the phase-sensitive nature of FWM creates comb lines and provides phase locking between them, as it is confirmed by the observation of stable trains of short pulses every round-trip.

To gain physical insight into the origin of the observed comb states, we focus on the first case of Figs. 1(a) and 1(b). Figure 2 shows the corresponding evolution of transient temporal and spectral profiles at various numbers of cavity round-trips. First, we note the amplification and compression of the intracavity modulated field, owing to the growth of new spectral components driven by FWM between the two pumps. This leads to the formation of a train of picosecond pulses with a local peak power higher than the cavity MI threshold [Fig. 2(b)]. Hence, the top of these pulses is subject to MI, which generates a fine multi-pulse structure [Fig. 2(c)]. The number of MI-induced pulses formed at the top of each pulse depends on the number of MI periods contained in the pulse profile above the MI threshold [four in Fig. 2(d) and only one in Fig. 2(f)]. In the bistable regime, the MI-induced pulses are reshaped into ultrashort soliton-like pulses, as shown in Fig. 2(h).

In Fig. 1(g), we may also observe the presence of a pair of peaks due to resonant radiations induced by the circulating soliton-like pulses. Such resonant dispersive waves (DWs) emerge at frequencies \(\omega\) that are quasi-phase-matched with the cavity soliton at \(\omega_0\) [18]. These frequencies can be simply estimated by assuming that the phase difference accumulated by the soliton and the DW across a single round-trip is equal to \(2\pi m\) (\(m\) is an integer) as follows:

\[
\sum_{j=1,2} \frac{\beta_2}{2} (\omega_0 - \omega_j)^2 l_j - \frac{\alpha \Delta}{2} = 2\pi m,
\]

(3)

where subscripts \(j = 1, 2\) indicate the type of fiber, namely HNL or SMF28, respectively. The parameter \(l\) corresponds to the fiber length. Here, we neglected the contribution of third-order dispersions. The predicted frequencies of fundamental

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**Fig. 1.** Numerical simulations of various comb states, obtained in the dual pumping configuration after 2000 cavity round-trips, as a function of the input power and cavity detuning. (a) and (b) \(\Delta = 1.1, P_0 = 0.15\) W; (c) and (d) \(\Delta = 1.1, P_0 = 0.3\) W; (e) and (f) \(\Delta = 1.6, P_0 = 0.12\) W; (g) and (h) \(\Delta = 2.15, P_0 = 0.2\) W. In sub-figures (b), (d), (f), and (h), the temporal red (black) traces indicate autocorrelation (pulse profiles). The blue curve in sub-figure (g) is the 415 fs sech-pulse fitting spectrum.
The generation of resonant DWs was also observed in other cases of Fig. 1 as soon as the spectral broadening overlaps their frequencies. DWs lead to the emergence of localized structures in the temporal domain. To illustrate the formation of the fundamental resonant DWs, we numerically calculated the time-frequency distribution of the optical field (see the spectrogram in Fig. 3) corresponding to the comb state of Figs. 1(g) and 1(h). At the point of maximum FWM-induced pulse compression, the symmetric spectral broadening overlaps with DW frequencies. In a transient state, the pulse train continuously sheds energy into the DWs. Ultimately, the circulating soliton-like pulses reach a steady state associated with the presence of DWs located on their weaker temporal sidelobes.

Figure 4 depicts our experimental setup based on a resonant passive fiber ring cavity made of a 26.5 m long segment of HNLF and 1.5-m of SMF28. A 90/10 coupler is used to form the fiber loop cavity; the resulting cavity finesse was estimated to be nearly 19. To pump the cavity, we split a CW laser at 1552.4 nm (linewidth < 1 kHz) into two beams. The first beam, labeled as the “control beam,” was directly injected into the cavity to fix and maintain the linear detuning $\varphi_0$ seen by the “pump beam” at each round-trip, with the help of a PID controller that finely tunes the laser wavelength. The second beam, labeled as the “pump beam,” was intensity modulated (at the null transmission point of the electro-optic modulator) to generate two pump lines of equal power, which are separated by a multiple of the fiber cavity FSR. (The separation between the two pumps was adapted to have both pumps on resonance.) Before injection into the cavity, the pump beam was again intensity modulated to generate 4 ns square pulses at 7.36 MHz repetition rate (corresponding to a cavity round-trip). Such pulse carving simultaneously enabled the increase of pump peak power and the removal of Brillouin backscattering within the cavity. The pulsed pump beam was then amplified by an erbium-doped fiber amplifier (EDFA), and launched into the cavity through the 90/10 coupler. To minimize their mutual interaction, the control and pump beams were counter-propagating. A careful control of input polarization state was obtained via polarization controllers to excite a neutral axis of the cavity fiber.

The temporal and spectral characterization of the comb states was provided by an intensity autocorrelator (with a temporal resolution of 10 fs and a full time window limited to 6 ps and 1 kHz). (m = ±1) and second-order (m = ±2) DWs from Eq. (3) are ±2.43 and ±3.40 THz, in excellent agreement with the observed positions in Fig. 1(g), thus validating their origin as resonant DWs.
80 ps) and a high-resolution (2.5 GHz) optical spectrum analyzer (OSA).

Figure 5 reports on experimental comb states obtained when the frequency spacing between the two pumps was fixed to approximately 80 GHz (i.e., the 7.36 MHz frequency and the pump spacing are commensurate so that both pumps are on resonance). It is worth mentioning that similar dynamics were also obtained for 40 GHz frequency spacing. Here, the measurements were performed for the same detunings as in the simulation: experimental results are found to be in good agreement with predictions. In the first three cases [Figs. 5(a) and 5(b), 5(c) and 5(d), 5(e) and 5(f)], the autocorrelation traces confirm the formation of a train of stable multi-pulse structures inside the cavity (two, four, and one pulse, respectively). The associated Kerr frequency combs are initially formed by multiple FWMs: subsequently, the MI leads to spectral shaping and induces the formation of multi-pulses within the 80 GHz temporal modulation. Note that the MI frequency is higher than the pump spacing, and the number of ultrashort localized structures forming within each period of the initial beating depends on the input pump power. When increasing the value of the linear detuning [see Figs. 5(e)–5(h)], the MI characteristics change so that only one pulse emerges from each FWM-induced modulation. In the bistable regime [Figs. 5(g) and 5(h)], we experimentally retrieve temporal and spectral signatures corresponding to the formation of a train of cavity soliton-like pulses driven by the modulated temporal field, as predicted by Fig. 1. (Note the discrepancy in pump power between the simulation and the experiment, maybe ascribed to the non-exact symmetrical detuning of the two pumps). For this regime, we numerically checked that a sudden switching of the modulated pump into a CW pump leads to the reshaping of generated pulses into proper cavity solitons [19]. Furthermore, the spectrum in Fig. 5(g) clearly exhibits four DW resonant peaks generated by the recirculating pulses, in agreement with our previous predictions.

In conclusion, we reported novel important aspects of the non-linear dynamics of Kerr frequency comb generation in a bichromatically pumped fiber ring cavity. We have shown that this particular configuration permits us to observe a large range of stable comb states, including periodic trains of nearly background-free cavity soliton-like pulses and associated multiple-order DWs.

**Funding.** Conseil Régional de Bourgogne; Agence Nationale de la Recherche (ANR) (ANR-12-BS04-0011); Partenariat Hubert Curien (PHC) Galilée/Programma Galileo; Ministero dell’Istruzione, dell’Università e della Ricerca (MIUR) (2012BFNW22).

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